Normative vs. Positive Models: Choice under Uncertainty

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- precursors date from 18^{th} century (D. Bernoulli)

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  - rather than just *assuming* EU maximization, vN-M showed that if decision maker (DM) satisfies basic, rather compelling assumptions, must act *as though* maximizing EU

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  - can understand complicated (and seemingly arbitrary) phenomenon (e.g., EU maximization) as *implication* of simple and less-arbitrary assumptions

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  - explained insurance markets well

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  - will discuss paradoxes of Allais, Ellsberg, Kahneman-Tversky

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- from (1), can assume $x_1 \succ x_2 \succ \ldots \succ x_n$ (labeling)

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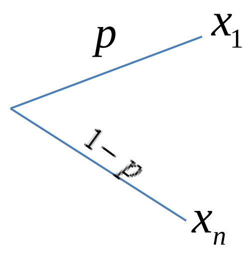

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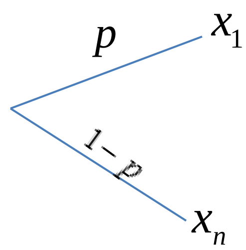

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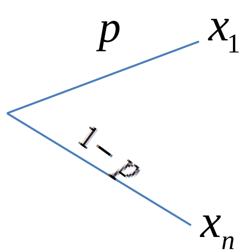

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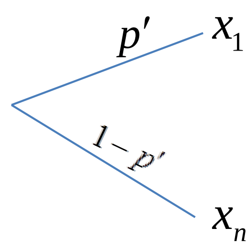

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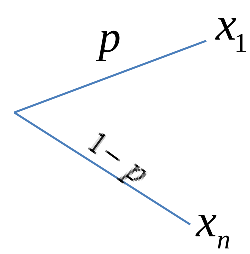
$\ell \sim$ 

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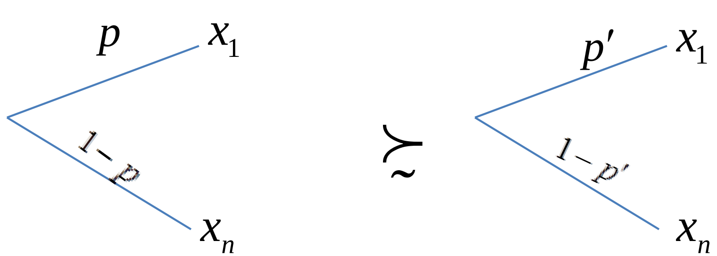

if and only if $p \geq p'$

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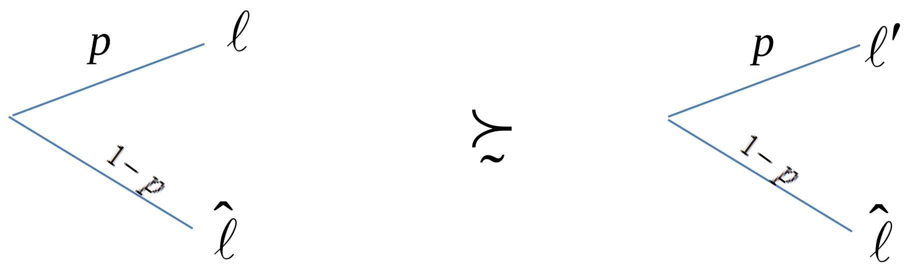

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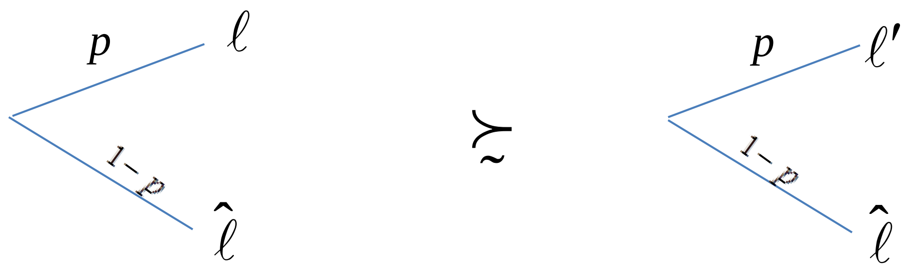

- only difference between two lotteries is:
  on right side, ℓ replaced by ℓ'

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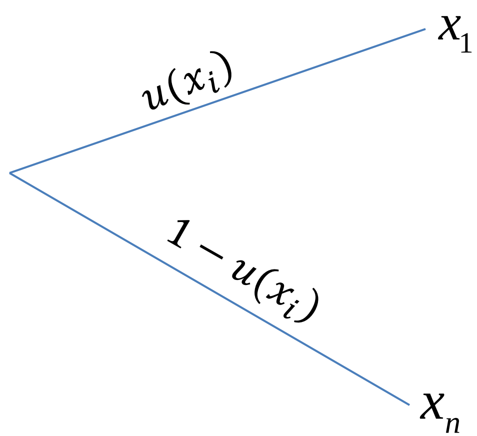

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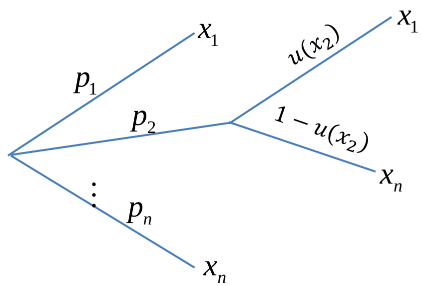

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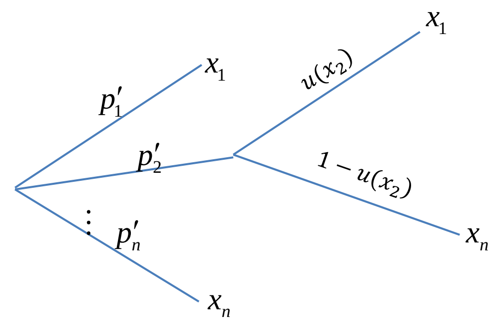

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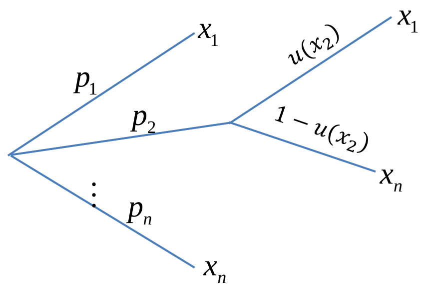

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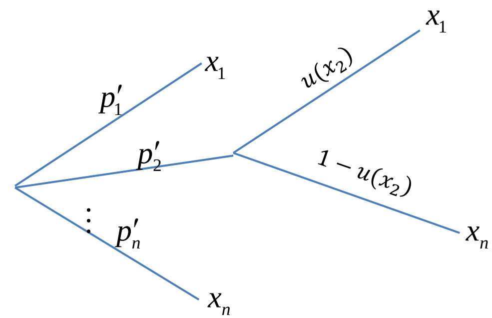

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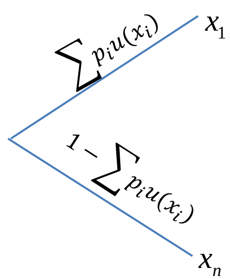

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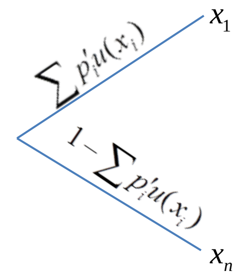

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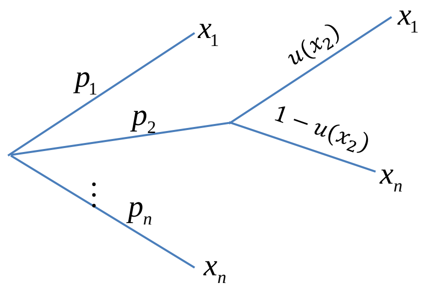

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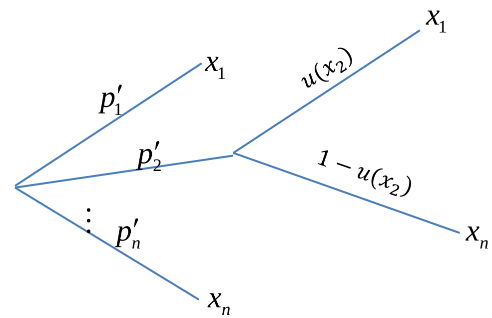

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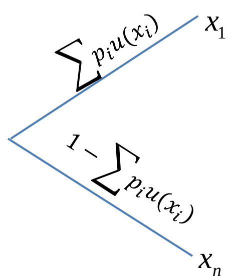

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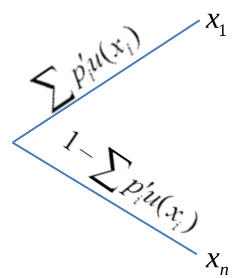

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monotonicity

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- risk aversion $\leftrightarrow$ utility function *u concave*

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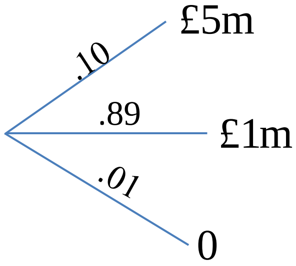

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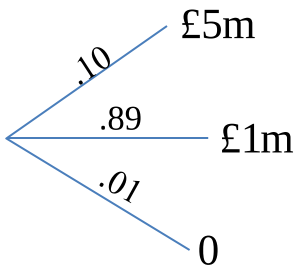

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- most people choose A

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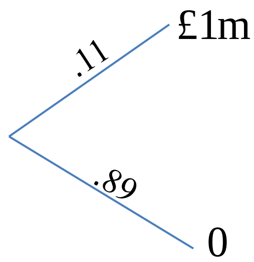

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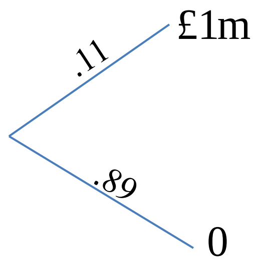

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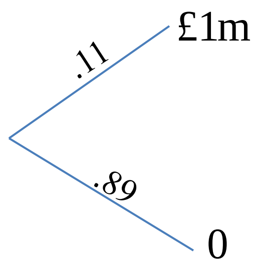

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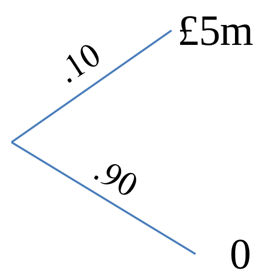
and



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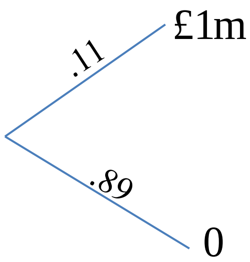

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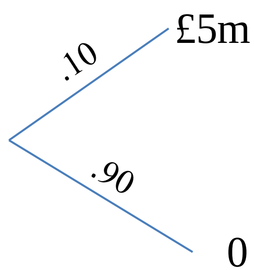

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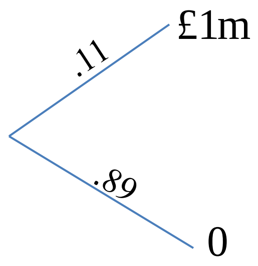

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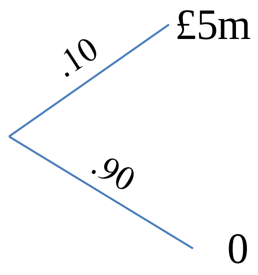
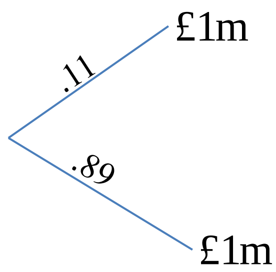


(D)

- most people choose D
- but choices A and D together violate EU!

- A can be rewritten as



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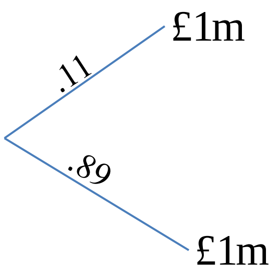
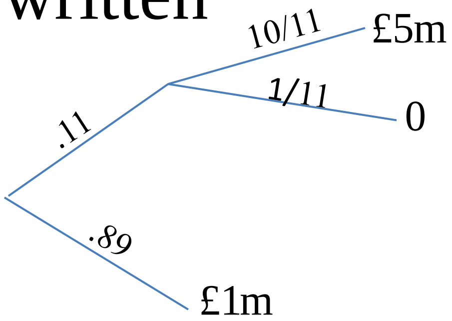


- B can be rewritten



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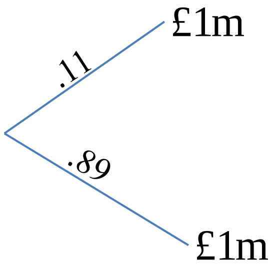

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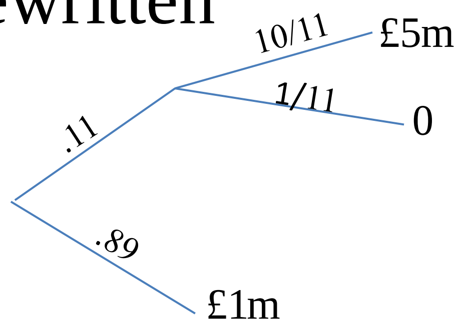
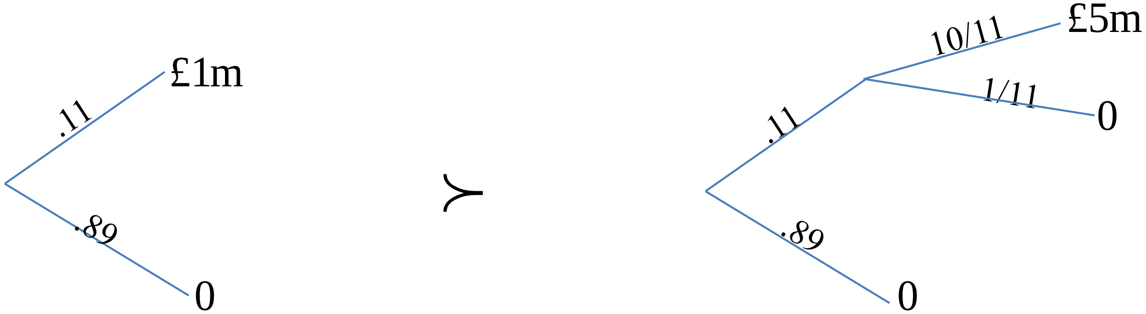


- but if A $\succ$ B, then independence axiom implies



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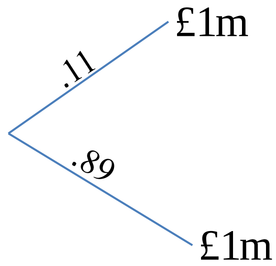

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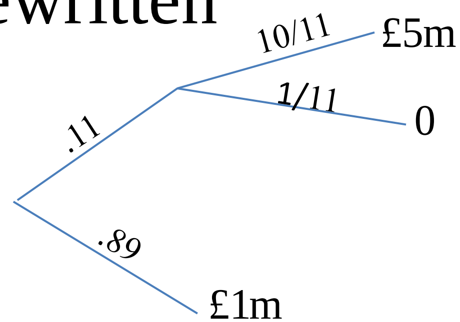

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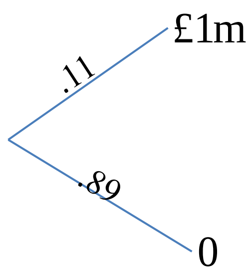
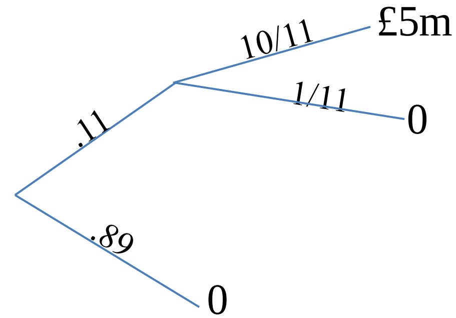


$\succ$



- so $C \succ D$

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- Savage (1954) reformulates vN-M axioms so that apply to case of *subjective* probability

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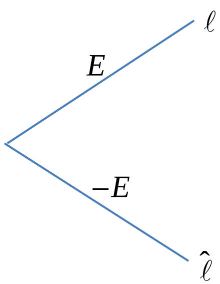
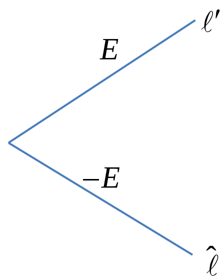


$\succsim$



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- same Ellsberg who leaked “Pentagon Paper” to press

Closed box containing 90 colored balls

	30	60	
	red	black	yellow
ℓ_1	£100	0	0
ℓ_2	0	£100	0
ℓ_3	£100	0	£100
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  - $\ell_4 \succ \ell_3 \rightarrow p(\text{black}) + p(\text{yellow}) > p(\text{red}) + p(\text{yellow})$

Kahneman-Tversky (1981)

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- casts doubt on whether can represent lottery unambiguously as $\ell = (p_1, \ldots, p_n)$

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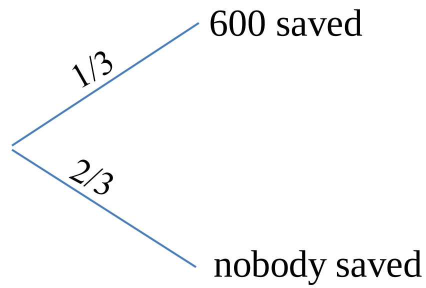

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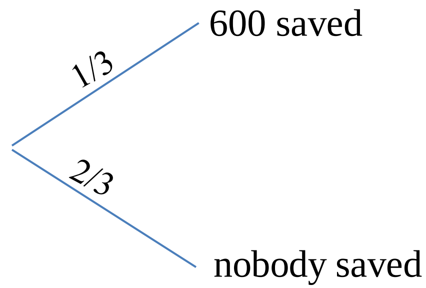

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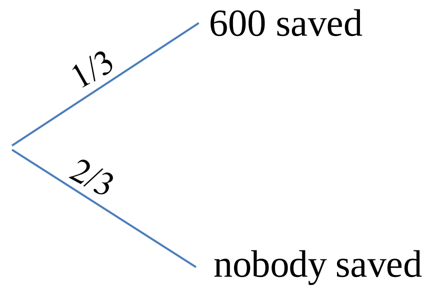

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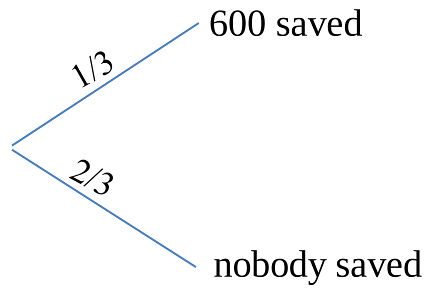

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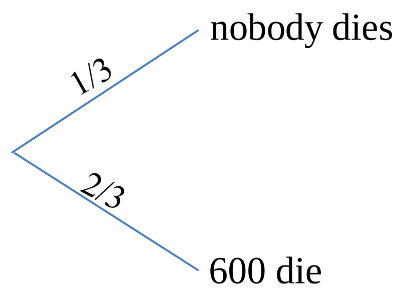

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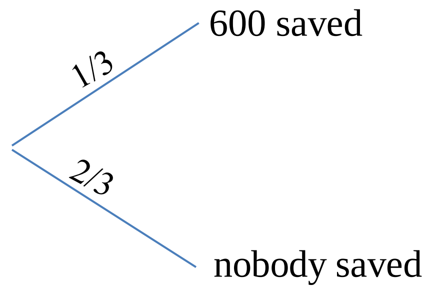

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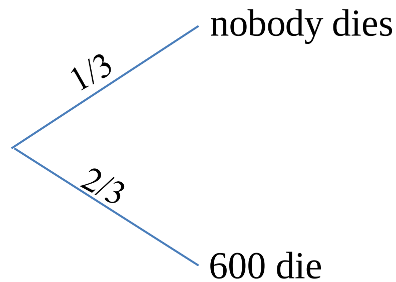

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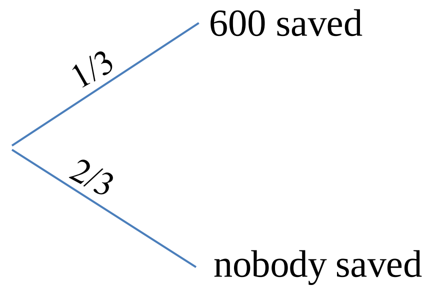

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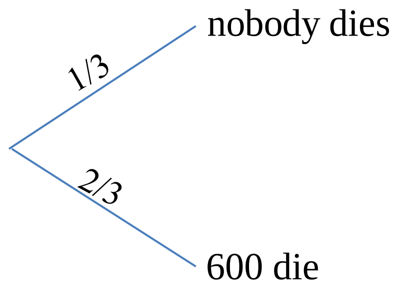

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- but A equivalent to C, B equivalent to D!

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